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**Buckling of Sandwich Cylinders under Combined Compression, Torsion,  
and Bending Loads**

CHI-TEH WANG, ROBERT J. VACCARO, AND DANIEL F. DE SANTO

College of Engineering, New York University

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BUCKLING OF SANDWICH CYLINDERS  
UNDER COMBINED COMPRESSION, TORSION,  
AND BENDING LOADS

By

Chi-Teh Wang \*\* Robert J. Vaccaro + Daniel F. DeSanto ++

New York University

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\*\* Professor of Aeronautical Engineering

+ Research Assistant

++ de la Cierva Fellow, Daniel Guggenheim School of Aeronautics

Summary

A theoretical investigation is carried out on the buckling of sandwich cylinders under combined compression, torsion, and bending loads. The governing differential equation is solved by using Galerkin's method. The interrelationship obtained between the critical loads is plotted in the form of non-dimensional interaction curves. In the limiting cases of axial compression alone, torsion alone, bending alone, and combined bending and axial compression, the results agree with those obtained previously.<sup>1-3</sup>

Symbols and Units

a = radius of cylinder to mid-plane, in.

C = hG, lbs. per in.

D = flexural stiffness of isotropic sandwich cylinder, in. lbs.

$$D = E t^2 / 2(1 - v^2)$$

E = Young's modulus for the face material lbs. per sq. in.

$$F = 2tE/a^2$$

G = shear modulus of the core material, lbs. per sq. in.

h = depth of isotropic sandwich plate measured between middle planes of faces, in.

I = moment of inertia of the sandwich cylinder about its diameter

l = length of cylinder, in.

M = bending moment, in. lbs.

m = number of half waves longitudinally

$N_x, N_y$  = resultant normal forces in x- and y-directions, lbs. per in.

$N_{xy}$  = resultant shearing force, lbs. per in.

$N_l$  = applied normal force in axial direction, lbs. per in.

n = number of full waves circumferentially

T = torsional moment, in. lbs.

t = thickness of the faces, in.

u, v, w = displacements in x-, y-, z-directions, respectively, of a point in middle surface of cylinder, in.

x, y, z = rectangular coordinates (Fig. 1)

v = Poisson's ratio for the face material

$\theta$  = circumferential coordinate

$$\nabla^4 = \text{operator}, \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)^2 = \left( \frac{\partial^4}{\partial x^4} + 2 \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \right) = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{a^2 \partial \theta^2} \right)^2$$

$$\nabla^8 = \text{operator}, \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)^4 = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{a^2 \partial \theta^2} \right)^4$$

$$\lambda = m\pi a/l$$

Introduction

In previous papers<sup>1-5</sup>, the senior author and his associates have investigated the buckling behavior of sandwich cylinders under axial compression, torsion, bending and combined bending and axial compression. The investigation is now extended to the buckling of sandwich cylinders under combined compression, torsion and bending loads. By solving Donnell's equation modified to include the effects of transverse shear for sandwich curved plates and shells using Galerkin's method, the buckling loads are calculated and are plotted in the form of interaction curves. In the limiting cases of bending alone, axial compression alone, torsion alone, and combined bending and axial compression, the results agree with those obtained previously.

### Formulation of The Problem

By assuming isotropic core material and neglecting the bending rigidity of the faces about their own middle surfaces, the equilibrium equations for an element of a sandwich cylinder have been derived in reference 4. By neglecting terms that were regarded as small by Donnell<sup>6</sup>, these equations can be reduced to a single equation in terms of the lateral deflection  $w$  only ..

$$D \nabla^8 w + (1 - \frac{D}{c} \nabla^2) \left[ \frac{2tE}{a^2} \frac{\partial^4 w}{\partial x^4} - \nabla^4 \right. \\ \left. (N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y}) \right] = 0 \quad (1)$$

This is the so-called Donnell's equation modified to include the effects of transverse shear for sandwich curved plates and shells first obtained by Stein and Mayers<sup>7</sup>.

In the case of sandwich cylinders under combined compression, torsion and bending loads, we have

$$N_x = -N_1 + (2Mat/I) \cos \theta, \\ N_{xy} = T/2\pi a^2, \quad N_y = 0 \quad (2)$$

where  $N_1$  is the force per unit length due to axial compression,  $M$  is the bending moment,  $I$  is the moment of inertia of the cylinder, and  $T$  is the torsional moment. With such a loading, equation (1) thus becomes

$$D \nabla^8 w + (1 - \frac{D}{c} \nabla^2) \left[ \frac{2tE}{a^2} \frac{\partial^4 w}{\partial x^4} - \nabla^4 \right. \\ \left. \left( -N_1 + \frac{2Mat}{I} \cos \theta \right) \frac{\partial^2 w}{\partial x^2} + \frac{T}{\pi a^3} \frac{\partial^2 w}{\partial x \partial \theta} \right] = 0 \quad (3)$$

where  $y$  is taken as  $a\theta$ .

### Galerkin's Method

Equation (3) may be solved by means of Galerkin's method as follows: we first assume the deflection  $w$  of the cylinder after buckling in the form of a series that satisfies the boundary conditions but with undetermined parameters. For long cylinders, however, the boundary conditions at the two ends become unimportant and we may assume the deflection series without any regard for the end conditions. For a cylinder under combined axial compression and bending,  $w$  may be assumed in the following form<sup>3</sup>,

$$w = \sin \frac{\pi x}{l} \sum_{n=0}^{\infty} A_n \cos n\theta \quad (4)$$

and for a cylinder under torsion only, the deflection after buckling is of the following form<sup>4</sup>,

$$\begin{aligned} w &= \sum_{n=1}^{\infty} B_n \sin \left( \frac{\pi x}{l} - n\theta \right) \\ &= \sum_{n=1}^{\infty} B_n \left( \sin \frac{\pi x}{l} \cos n\theta - \cos \frac{\pi x}{l} \sin n\theta \right). \end{aligned} \quad (5)$$

Guided by these expressions, we shall assume the deflection of the cylinder in the case of combined loading as follows:

$$w = \sin \frac{\pi x}{l} \sum_{n=0}^{n_1} A_n \cos n\theta + \cos \frac{\pi x}{l} \sum_{n=1}^{n_1} B_n \sin n\theta. \quad (6)$$

With  $w$  expressed in a proper series, we shall next substitute this series into (3). If the expression (6) happens to be the exact solution of equation (3), after substitution equation (3) will be identically equal to zero. In general, this will not be so and the resulting expression will be a function of  $x$  and  $\theta$  which we shall denote by  $Q$ . Galerkin's

equations for the determination of the coefficients  $A_n$  and  $B_n$  are

$$\int_0^{2\pi} \int_0^l Q \sin \frac{m\pi x}{l} \cos n\theta \sin \theta dx = 0 \quad (7)$$

$$\int_0^{2\pi} \int_0^l Q \cos \frac{m\pi x}{l} \sin n\theta \sin \theta dx = 0 \quad (8)$$

which, when written out, become

$$\begin{aligned} & \int_0^{2\pi} \int_0^l \left( D \nabla^8 w + F \frac{\partial^4 w}{\partial x^4} - \frac{DF}{C} \nabla^2 \frac{\partial^4 w}{\partial x^4} \right) \sin \frac{m\pi x}{l} \cos n\theta \sin \theta dx \\ & + H_1 \int_0^{2\pi} \int_0^l \left( \nabla^4 \frac{\partial^2 w}{\partial x^2} - \frac{D}{C} \nabla^6 \frac{\partial^2 w}{\partial x^2} \right) \sin \frac{m\pi x}{l} \cos n\theta \sin \theta dx \\ & - \frac{2Mat}{I} \int_0^{2\pi} \int_0^l \left( \nabla^4 \frac{\partial^2 w}{\partial x^2} - \frac{D}{C} \nabla^6 \frac{\partial^2 w}{\partial x^2} \right) \sin \frac{m\pi x}{l} \cos n\theta \cos \theta \sin \theta dx \\ & - \frac{T}{\pi a^3} \int_0^{2\pi} \int_0^l \left( \nabla^4 \frac{\partial^2 w}{\partial x \partial \theta} - \frac{D}{C} \nabla^6 \frac{\partial^2 w}{\partial x \partial \theta} \right) \sin \frac{m\pi x}{l} \cos n\theta \sin \theta dx = 0, \end{aligned} \quad (9)$$

in which  $F = 2tE/a^2$ , and

$$0 \int_0^{2\pi} \left( D \nabla^2 w + F \frac{\partial^4 w}{\partial x^4} - \frac{DF}{C} \nabla^2 \frac{\partial^4 w}{\partial x^4} \right) \cos \frac{m\pi x}{l} \sin n\theta \sin \theta dx$$

$$+ N_1 \int_0^{2\pi} \left( \nabla^4 \frac{\partial^2 w}{\partial x^2} - \frac{D}{C} \nabla^6 \frac{\partial^2 w}{\partial x^2} \right) \cos \frac{m\pi x}{l} \sin n\theta \sin \theta dx$$

$$- \frac{2Mat}{I} \int_0^{2\pi} \int_0^l \left( \nabla^4 \frac{\partial^2 w}{\partial x^2} - \frac{D}{C} \nabla^6 \frac{\partial^2 w}{\partial x^2} \right) \cos \frac{m\pi x}{l} \sin n\theta \cos \theta \sin \theta dx$$

$$- \frac{T}{\pi a^3} \int_0^{2\pi} \int_0^l \left( \nabla^4 \frac{\partial^2 w}{\partial x \partial \theta} - \frac{D}{C} \nabla^6 \frac{\partial^2 w}{\partial x \partial \theta} \right) \cos \frac{m\pi x}{l} \sin n\theta \sin \theta dx = 0.$$

(10)

Substituting the assumed function  $w$  into these equations and carrying out the integration, we obtain

$$\left[ \frac{D(\lambda^2 + n^2)^4}{a^8} + \frac{F \lambda^4}{a^4} + \frac{DF \lambda^4 (\lambda^2 + n^2)}{Ca^6} \right. \\ \left. \frac{N_1 \lambda^2 (\lambda^2 + n^2)^2}{a^6} - \frac{N_1 D \lambda^2 (\lambda^2 + n^2)^3}{Ca^8} \right] A_n$$

$$+ \left[ \frac{m \lambda (\lambda^2 + n^2)^2}{\pi a^8} + \frac{m n \lambda (\lambda^2 + n^2)^3}{\pi C a^{10}} \right] B_n \\ + \left[ \frac{Mat \lambda^2 (\lambda^2 + n^2)^2}{Ia^6} + \frac{Mat D \lambda^2 (\lambda^2 + n^2)^3}{ICa^8} \right] (A_{n-1} + A_{n+1}) = 0,$$

(11)

and

$$\begin{aligned}
 & \left[ \frac{D(\lambda^2 + n^2)^4}{a^8} + \frac{r\lambda^4}{a^4} + \frac{DF\lambda^4(\lambda^2 + n^2)}{Ca^6} \right. \\
 & \quad \left. - \frac{N_1\lambda^2(\lambda^2 + n^2)^2}{a^6} - \frac{N_1D\lambda^2(\lambda^2 + n^2)^3}{Ca^8} \right] B_n \\
 & + \left[ \frac{m\lambda(\lambda^2 + n^2)^2}{\pi a^8} + \frac{mDn\lambda(\lambda^2 + n^2)^3}{\pi Ca^{10}} \right] A_n \\
 & + \left[ \frac{Mat\lambda^2(\lambda^2 + n^2)^2}{Ia^6} + \frac{MatD\lambda^2(\lambda^2 + n^2)^3}{ICa^8} \right] (B_{n-1} + B_{n+1}) = 0. \tag{12}
 \end{aligned}$$

Instead of working with these two equations, it is found convenient to combine them into the following ones. Thus, by subtracting equation (12) from (11), we obtain

$$\begin{aligned}
 & \left[ \frac{D(\lambda^2 + n^2)^4}{a^8} + \frac{r\lambda^4}{a^4} + \frac{DF\lambda^4(\lambda^2 + n^2)}{Ca^6} \right. \\
 & \quad \left. - \frac{N_1\lambda^2(\lambda^2 + n^2)^2}{a^6} - \frac{N_1D\lambda^2(\lambda^2 + n^2)^3}{Ca^8} \right. \\
 & \quad \left. - \frac{m\lambda(\lambda^2 + n^2)^2}{\pi a^8} - \frac{mDn\lambda(\lambda^2 + n^2)^3}{\pi Ca^{10}} \right] K_n \\
 & + \left[ \frac{Mat\lambda^2(\lambda^2 + n^2)^2}{Ia^6} + \frac{MatD\lambda^2(\lambda^2 + n^2)^3}{ICa^8} \right] (K_{n-1} + K_{n+1}) = 0, \tag{13}
 \end{aligned}$$

in which  $\lambda = \pi\tau a/l$ ,  $K_n = A_n - B_n$ , and in which  $2K_0$  is to be substituted for  $K_0$  at  $n = 1$ ;  $K_n = 0$  when  $n > n_1$ . When equations (11) and (12) are added, we have

$$\begin{aligned}
 & \left[ \frac{D(\lambda^2 + n^2)^4}{a^8} + \frac{F\lambda^4}{a^4} + \frac{DF\lambda^4(\lambda^2 + n^2)}{Ca^6} \right. \\
 & \quad - \frac{N_1\lambda^2(\lambda^2 + n^2)^2}{a^6} - \frac{N_1D\lambda^2(\lambda^2 + n^2)^3}{Ca^8} \\
 & \quad \left. + \frac{Tn\lambda(\lambda^2 + n^2)^2}{\pi a^8} + \frac{TDn\lambda(\lambda^2 + n^2)^3}{\pi Ca^{10}} \right] K'_n \\
 & + \left[ \frac{Mat\lambda^2(\lambda^2 + n^2)^2}{Ia^6} + \frac{MatD\lambda^2(\lambda^2 + n^2)^3}{ICa^8} \right] (K'_{n-1} + K'_{n+1}) = 0,
 \end{aligned} \tag{14}$$

in which  $K'_n = A_n + B_n$ . Again at  $n = 1$ ,  $2K'_0$  is to be substituted for  $K'_0$  and  $K'_n = 0$  when  $n > n_1$ .

The substitution of  $n$  from 0 to  $n_1$  in equations (13) and (14) results in  $2n_1 + 1$  simultaneous algebraic equations for  $2n_1 + 1$  unknowns. One solution to these equations is of course, the trivial one, namely,

$$K_n = K'_n = 0 \quad (n = 0, 1, 2, 3, \dots, n_1).$$

The non-trivial solution is carried out in the following section.

### Determination of the Buckling Loads

To find the non-trivial solutions of equations (13) and (14) in their present forms is a very difficult problem. However, from the previous investigations<sup>1,2,3</sup> it was found that the minimum buckling loads occur at  $\lambda = \infty$ . Dividing equations (13) and (14) by  $\lambda^8$ , and remembering that  $n_1$  may also be very large, we shall drop all terms containing  $\lambda$  to a power greater than zero in the denominator but keep terms containing the ratio  $n_1/\lambda$ . Equations (13) and (14) thus become

$$(1 + \frac{n^2}{\lambda^2} - \frac{N_1}{C} - \frac{T}{\pi a^2 C} \frac{n}{\lambda}) K_n + \frac{Mat}{IC} (K_{n-1} + K_{n+1}) = 0 \quad (15)$$

$$(1 + \frac{n^2}{\lambda^2} - \frac{N_1}{C} + \frac{T}{\pi a^2 C} \frac{n}{\lambda}) K'_n + \frac{Mat}{IC} (K'_{n-1} + K'_{n+1}) = 0 \quad (16)$$

After dividing these equations through by  $Mat/IC$ , we have

$$K_{n+1} + L_n K_n + K_{n-1} = 0, \quad (17)$$

$$K'_{n+1} + L'_n K'_n + K'_{n-1} = 0 \quad (18)$$

in which

$$L_n = \frac{IC}{Mat} \left( 1 + \frac{n^2}{\lambda^2} - \frac{N_1}{C} - \frac{T}{\pi a^2 C} \frac{n}{\lambda} \right), \quad (19)$$

$$L'_n = \frac{IC}{Mat} \left( 1 + \frac{n^2}{\lambda^2} - \frac{N_1}{C} + \frac{T}{\pi a^2 C} \frac{n}{\lambda} \right) \quad (20)$$

Following the examples worked out in Reference 3, one may attempt to solve equations (17) and (18) as finite-difference equations. This, however, was not found possible. The reason is that equations (17) and (18) are now finite-difference equations with variable coefficients and the solutions of such equations are difficult mathematical problems. Instead we shall solve the problem in the following manner.

Let us first investigate the magnitude of  $n_1$ . If  $n_1$  is small compared to  $\lambda$  then  $n_1/\lambda \rightarrow 0$  as  $\lambda \rightarrow \infty$  and equations (17) and (18) reduce to the governing equations in the case of combined compression and bending. In order that the torsional load may have any effect on the buckling phenomena,  $n_1$  must be also a large number so that the ratio  $n_1/\lambda$  may remain finite. For any  $n$  close to  $n_1$ , say,  $n = n_1-1, n_1-2, \dots, n_1-q$ , ( $q \ll n_1$ ), it is obvious that such terms as  $1/\lambda, 2/\lambda, \dots, q/\lambda$  vanish as  $\lambda \rightarrow \infty$ . Thus, for the equations in which  $n = n_1, n_1-1, n_1-2, \dots, n_1-q$ , the following relations hold:

$$\begin{aligned} L_{n_1} &= L_{n_1-1} = L_{n_1-2} = \dots = L_{n_1-q} \\ &= \frac{IC}{Mat} \left( 1 + \frac{n_1^2}{\lambda^2} \right) - \frac{N_1}{C} - \frac{T}{\pi a^2 C} \frac{n_1}{\lambda} \\ &= L \end{aligned} \quad (21)$$

Therefore, if in the series for  $w$ , we take the summation of terms from  $n_1-q$  to  $n_1$ , the system of equations will be as follows:

$$\left. \begin{aligned} LK_{n_1} + K_{n_1-1} &= 0 \\ K_{n_1} + LK_{n_1-1} + K_{n_1-2} &= 0 \\ \dots \\ K_{n_1-q+1} + LK_{n_1-q} &= 0 \end{aligned} \right\} \quad (22)$$

and

$$\left. \begin{aligned} L'K'_{n_1} + K'_{n_1-1} &= 0 \\ K'_{n_1} + L'K'_{n_1-1} + K'_{n_1-2} &= 0 \\ \dots \\ K'_{n_1-q+1} + L'K'_{n_1-q} &= 0 \end{aligned} \right\} \quad (23)$$

$$\text{where } L' = \frac{IC}{Mat} \left( 1 + \frac{n_1^2}{\lambda^2} - \frac{N_1}{C} + \frac{T}{\pi a^2 C} \frac{n_1}{\lambda} \right)$$

To obtain the non-trivial solution of (22) we set the determinant of the coefficients  $K_n$  equal to zero, namely,

$$\begin{vmatrix} L & 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 1 & L & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & L & 1 & 0 & \dots & 0 & 0 \\ & & & \ddots & & & & \\ & & & \ddots & & & & \\ & & & \ddots & & & & \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & L \end{vmatrix} = 0 \quad (24)$$

Equation (24) can be rewritten as

$$\begin{aligned} L - \frac{1}{L - \frac{1}{L - \frac{1}{L - \dots}}} &= 0 \\ L - \frac{1}{L - \frac{1}{L - \frac{1}{L - \frac{1}{L - \dots}}}} &= 0 \\ &\vdots \\ L - \frac{1}{L - \frac{1}{L - \frac{1}{L - \frac{1}{L}}}} &= 0 \end{aligned} \quad (25)$$

By taking more and more terms in the expression for  $w$ , we have successively larger and larger continued fractions in (25) and in each case the roots  $L$  are found.

The question now arises as to which of the roots obtained in solving equation (25) is the one which should be used in the final analysis. This is easily determinable from relation (21), which, when rewritten, becomes

$$M = \frac{IC}{Lat} \left( 1 + \frac{n_1^2}{\lambda^2} - \frac{N_1}{C} - \frac{T}{\pi a^2 C} \frac{n_1}{\lambda} \right) \quad (26)$$

Obviously the lowest value of the critical bending moment  $M$  occurs for the largest value of  $L$ ; therefore, in solving equation (25), the largest root obtained in each case is the one selected.

It is noted (Fig. 2) that the largest root of equation (26) approaches the value of 2.00 as larger and larger continued fractions are considered. Therefore the value  $L = 2.00$  is taken as the solution. Equation (26) therefore becomes, after dividing through by  $IC/L^2$ ,

$$\frac{2Mat}{IC} = 1 + \frac{n_1^2}{\lambda^2} - \frac{N_1}{C} - \frac{T}{\pi a^2 C} \frac{n_1}{\lambda}. \quad (27)$$

In order to find the lowest possible value of the term  $2Mat/IC$ , it is necessary to minimize the right-hand side of (27) with respect to the variable  $n_1/\lambda$ . It may be pointed out that  $n_1/\lambda$  actually takes on only discrete values on account of the integral character of  $n_1$  and of  $\lambda (= \pi a/l)$ . However, where  $\lambda$  is very large (as it is in this case), it is possible to consider  $n_1/\lambda$  as a continuous function in the minimizing process.

Having established that it is permissible to consider  $n_1/\lambda$  as a continuous variable, let us minimize the right-hand side of (27) by setting

$$\frac{\partial}{\partial(n_1/\lambda)} \left( 1 + \frac{n_1^2}{\lambda^2} - \frac{N_1}{C} - \frac{T}{\pi a^2 C} \frac{n_1}{\lambda} \right) = 0. \quad (28)$$

Hence

$$2 \frac{n_1}{\lambda} - \frac{T}{\pi a^2 C} = 0$$

or

$$\frac{n_1}{\lambda} = \frac{T}{2\pi a^2 C} \quad (29)$$

Consequently, equation (27) becomes

$$\frac{2Mat}{IC} = 1 - \frac{N_1}{C} - \left( \frac{T}{2\pi a^2 C} \right)^2 \quad (30)$$

or

$$\frac{T}{2\pi a^2 C} = \pm \sqrt{1 - \frac{N_1}{C} - \frac{2Mat}{IC}} \quad (31)$$

Exactly the same result is obtained if we solve for the non-trivial solution for equation (23).

In the case where the cylinder is under axial compression alone,  $M = T = 0$ , and equation (31) becomes

$$\pm \sqrt{1 - \frac{N_1}{C}} = 0, \quad (32)$$

or

$$N_1 = C$$

Equation (32) is exactly the one obtained in Reference 1.

In the case where the cylinder is under torsional loads alone,  $M = N_1 = 0$  and equation (31) becomes

$$\frac{T}{2\pi a^2 C} = \pm 1, \quad (33)$$

or

$$\frac{N_1}{K_f} = \pm C.$$

Equation (33) is the identical equation obtained in Reference 2.

In the case where the cylinder is under bending moments alone, equation (31) becomes

$$\pm \sqrt{1 - \frac{2Mat}{IC}} = 0, \quad (34)$$

or

$$\frac{2Mat}{I} = C.$$

Equation (34) is exactly the one obtained in Reference 3.

If we define the stress ratios  $R_C$ ,  $R_B$ ,  $R_T$  according to the following formulas,

$$R_C = \frac{N_1}{C} = \frac{\text{Critical compressive stress}}{\text{Buckling stress under compression alone}}$$

$$R_B = \frac{2^t}{IC} = \frac{\text{Critical bending moment}}{\text{Buckling moment under bending alone}}$$

$$R_T = \frac{T}{2\pi a^2 C} = \frac{\text{Critical torsional moment}}{\text{Buckling moment under torsion alone}}$$

then equation (31) may be written as

$$R_T = \pm \sqrt{1 - (R_B + R_C)}. \quad (35)$$

The interrelationship between compression, bending, and torsional stress ratios given by equation (35) is plotted for engineering use in Figs. 3 and 4. Once any two stress ratios are specified, the buckling value of the remaining stress ratio can be determined graphically from these curves.

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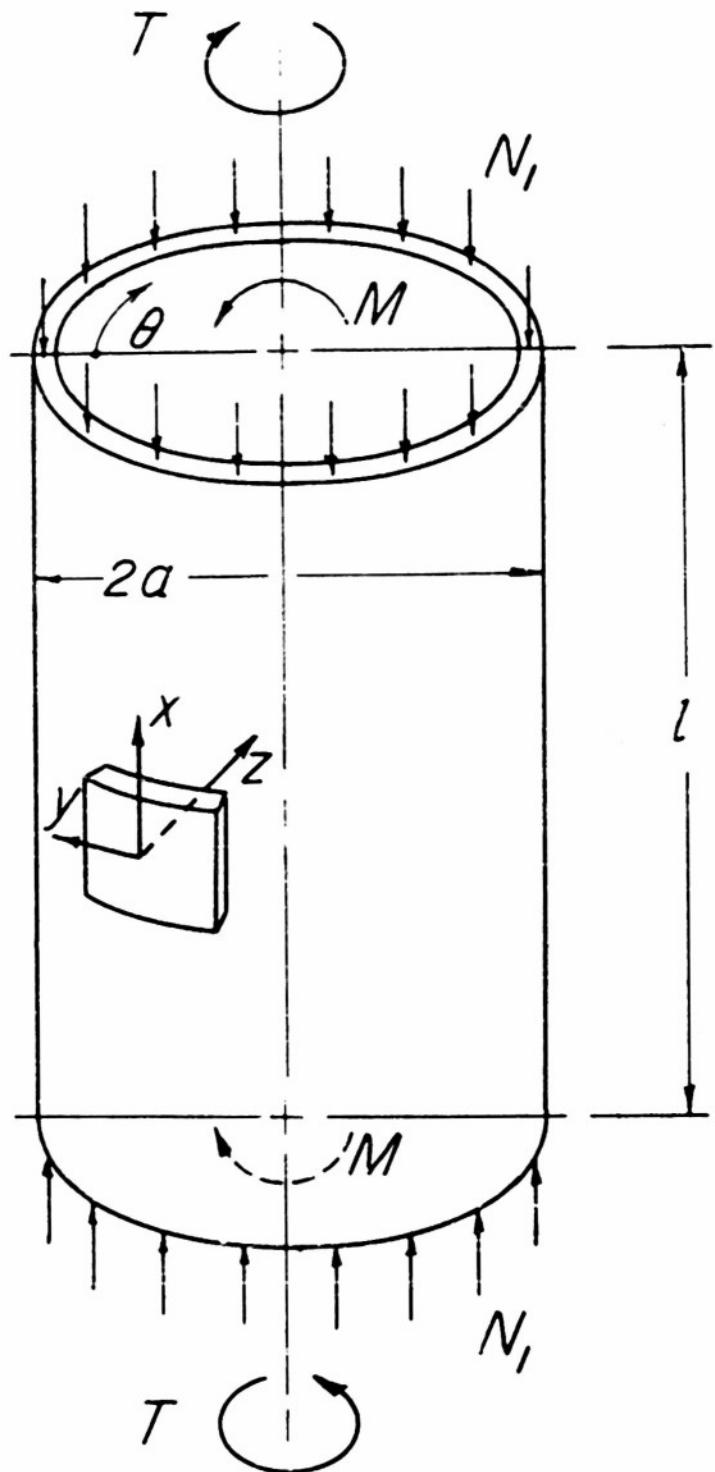


FIGURE I. SANDWICH CYLINDER  
CONFIGURATION AND LOADING

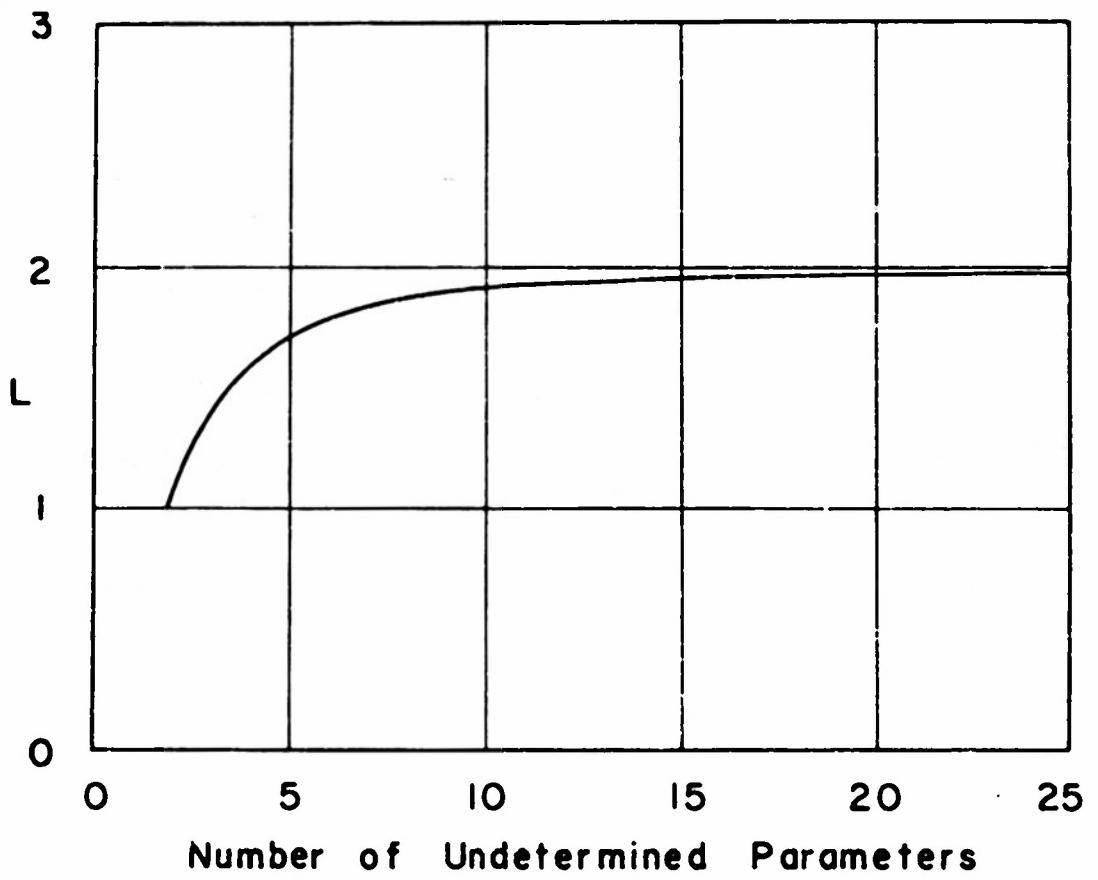


FIGURE 2. RESULT OF EVALUATING L  
BY METHOD OF SUCCESSIVE APPROXIMATIONS

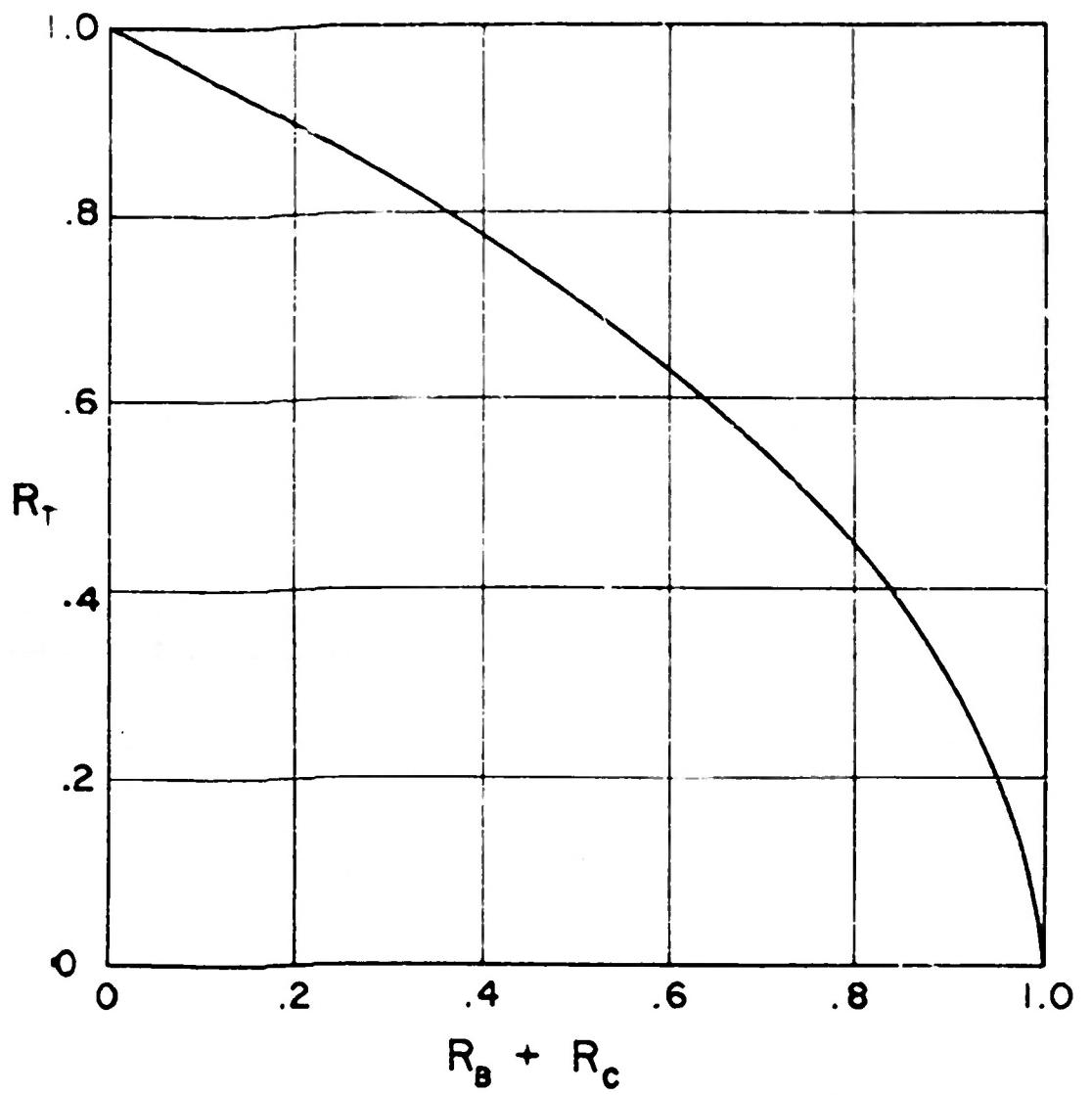


FIGURE 3. INTERACTION CURVE: TORSION STRESS  
RATIO vs. SUM OF BENDING AND COMPRESSION  
STRESS RATIOS

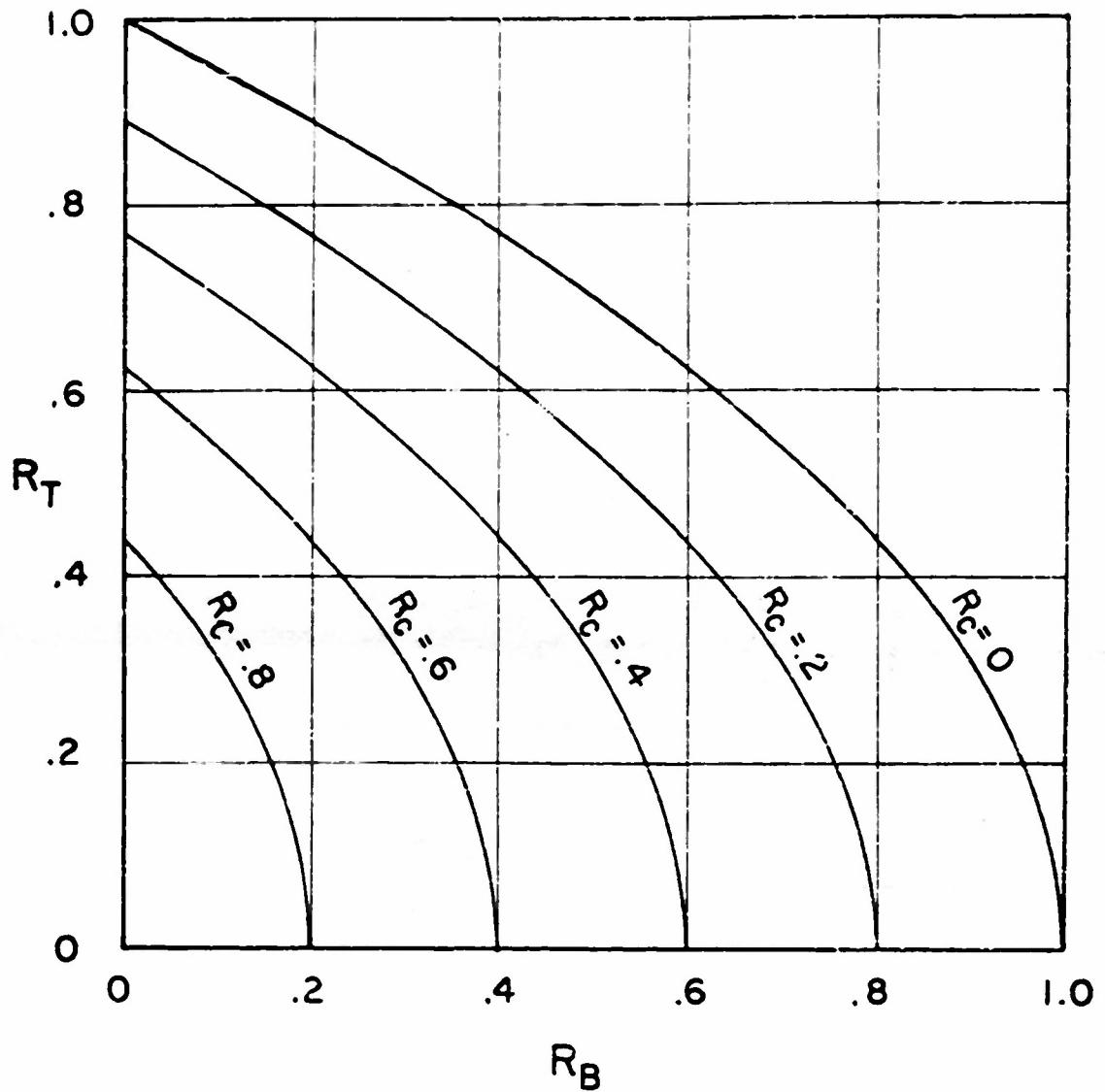


FIGURE 4. INTERACTION CURVES:  
COMBINED LOADING